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PATENT

Paper No.

File: Greene-P1-03

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

Inventor	:	REES, Frank L.
Serial No.	:	10/722,648
Filed	:	November 25, 2003
For	:	Gauss-Rees Parametric Ultrawideband System
Group Art Unit	:	3662
Examiner	:	LOBO, IAN J

MS: AAF
Honorable Commissioner of Patents
P.O. Box 1450
Alexandria, VA 22313-1450

<u>RESPONSE</u>

S I R :

In response to the Office Action mailed 02/27/2007 in the above-referenced

patent application, please reconsider the application in view of the following remarks.

I.   **RESPONSE**

Claims 1, 61, and 64-65 are pending, and all pending claims have been found allowable except for claim 1.

Claim 1 has been rejected pursuant to 35 U.S.C. Sec. 103. The Examiner contends that claim 1 is obvious over Sen et al., in view of Rees, as is more precisely set forth in paragraph 3 of the Office Action. Claims 61, and 64-65 have been found allowable if rewritten as incorporating claim 1.

The rejection is respectfully traversed and reconsideration is requested. Definitional rebuttal evidence is enclosed herewith.

The rejection is improper for the reasons set out in the prior Response and more particularly stated hereinafter.

1.   All claim elements have not been shown in the cited art.

Applicant's Response notes that (1) all claim elements have not been shown in the cited art.

In the Office Action dated February 27, 2007, the Examiner states: "applicant argues that the Sen et al. patent does not disclose or show the claimed "secondary wavelet... This is not agreed with. Sen et al. discloses "backscattering..."

Backscattering does not disclose a secondary wavelet, e.g, in Sen et al.

In its plain and ordinary meaning with regard to acoustics, "backscattering" refers to:

> the deflection of radiation or particles by scattering through angles greater than
> 90 (degrees) with reference to the original direction of travel.

See, for example, the enclosed definition from CHAMBERS: *Dictionary of Science and Technology* (1999); see also McGraw-Hill Dictionary of Scientific and Technical Terms (1984) (also enclosed).

In its plain and ordinary meaning with regard to acoustics, "wavelets" refers to:

mathematical functions that cut up data into different frequency components, and then study each component with a resolution matched to its scale. They have advantages over traditional Fourier methods in analyzing physical situations where the signal contains discontinuitie*s and sharp spikes*, as well as being compactly constrained in time duration.

See, for example, the enclosed - which originally appeared in The Institute of Electrical and Electronics Engineers) (IEEE) Computational Science and Engineering, Summer 1995, vol. 2, num. 2, published by the IEEE Computer Society, 10662 Los Vaqueros Circle, Los Alamitos, CA 90720, USA, TEL +1-714-821-8380, FAX +1-714-821-4010, and can be found on the Web as www.amara.com/IEEEwave/IEEEwavelet.html.

It would seem from the plain and ordinary meaning of the terms that the Examiner's contended "backscattering" **has no bearing whatsoever** on the claimed secondary wavelet... etc.

35 U.S.C. Sec. 132 requires that the PTO provide "the reasons for such rejection... together with such information as may be useful in judging the propriety of continuing prosecution...." There is no mention whatsoever of the claimed secondary wavelet in cited art cited in the Office Action. Thus, the rejection is premised on the Examiner's contention that backscattering in Sen et al. discloses the claimed secondary wavelet. Accordingly, pursuant to 35 U.S.C. Sec. 132, the Examiner is required to explain how, pursuant to his rejection, backscattering discloses the secondary wavelet. Further, the Examiner's declaration or affidavit to establish the contention is required.

For reference, in the instant application, secondary wavelet information is received by means of 18 in Figure 30 and Figure 31, and processed in Figure 31 at boxes 38 and/or 40, with the main processor being 42. See also Applicant's specification as it refers to these items. The Examiner's 132 explanation and the Examiner's declaration should identify the *particular* components of Sen et al. that are contended as:

receiving the secondary wavelet produced by the nonlinear effect; and

processing the received secondary wavelet in identifying the object.

Further, the Examiner's attention is drawn to instant application para. 146, wherein the last sentence states:

> If the desire is to produce clean seismic, multi-channel data stacking or to employ spectroscopic analysis for discerning material-specific additional spectral components (that are *induced by nonlinear interaction within or inelastic scattering from concealed material*), a clean Secondary Wavelet energy spectrum is important.

(Emphasis added.) The Examiner's 132 explanation and the Examiner's declaration should explain by what means his contended secondary wavelet is produced by the nonlinear effect in Sen et al.

Applicant maintains that the applied references fail to disclose all expressly claimed elements or limitations, and the rejection is improper for want of a *prima facie* showing of unpatentability.

2. The contended combination of teachings would render the cited art inoperable for their respective purposes.

Applicant's Response notes that (2) the contended combination of teachings would render the cited art inoperable for their respective purposes.

In the Office Action dated February 27, 2007, the Examiner states: "This is also not convincing since the mere substitution of one non-linear acoustic source (Rees) for another (Sen et al.) would not render inoperable the system of Sen et al.

Rees is an optical detection system and Sen et al. is not.

Rees uses a laser to detect sound generated by weather: as regards detection, the acoustic source in Rees is inclement weather (Col. 1, line 39, etc.).

Sen et al. uses an acoustic pulse emitter and corresponding sensors.

The non-overlapping PTO classes of Rees and Sen et al. evidence that these patents involve different technologies, and presumably the Examiner appreciates that, on the face of it, using a laser to optically detect weather (Rees) is not readily modifiable into a system

using sound (from an acoustic pulse emitter Sen et al.) to detect land mines underground.

In the Office Action, the Examiner has pointed to Col. 15, line 58 - Col. 16, line 4

in Rees, where Rees refers to

> using a non-linearly generated and radial range focused acoustic sawtooth wave
> to create an acousto-optic mirror (AOM) acting as a retroreflector. Sufficient
> acoustic enhancement to crate a shock front to a nonlinear acoustic sawtooth
> wave is brought about by transmitting a synthetic spectrum waveform using a
> multiple set of phase locked, pulsed acoustic carrier waveforms each emitted
> from individual projectors in a large array of loudspeakers. Constructive
> interference occurs when these acoustic pulses come together to add
> coherently... the each of these discontinuities act as an optical mirror.

So Rees is still a system for optical detection of inclement weather that can involve "transmitting

a synthetic spectrum waveform using a multiple set of phase locked, pulsed acoustic carrier

waveforms each emitted from individual projectors in a large array of loudspeakers" to obtain

"constructive interference" that acts "as an optical mirror."

The Examiner contends that "the mere substitution of one non-linear source

(Rees) for another (Sen et al.) would not render inoperable the system of Sen et al. However,

an acousto-optic mirror would not function in the granular bed of Sen et al. - e.g., for detecting

land mines. That is, Rees's optical mirror is to redirect a laser beam, which of course seemingly

could not possibly function in the ground in Sen et al. If the rejection is maintained, this is for

the Examiner to explain.

Pursuant to 35 U.S.C. Sec. 132 the Examiner is required to explain how the non-

linear source of Rees that is adapted to produce "constructive interference" and that acts "as an

optical mirror" would not render inoperable the system of Sen et al. Operated to cause

"constructive interference" as taught by Rees, the non-linear source in Rees would presumably

"interfere" with the acoustic detection of land mines in Sen et al. So operated, the creation of an

optical mirror would also seem to render Sen et al. nonfunctional for its purpose, e.g., detecting

underground land mines. The Examiner's 132 explanation and information is required.

Further, the Examiner's contention is a contention of fact on a point where the

5

PTO bears the burden of proof, and affidavit or declaration on the contention of fact, are required.

Applicant maintains that the contended combination of teachings would render the cited art inoperable for their respective purposes, and the rejection is improper for want of a *prima facie* showing of unpatentability.

3. The contended modification or substitution of the teachings to reach the claimed invention would change the principles of operation of the devices shown in the respective cited art.

Applicant's Response notes that (3) the contended modification or substitution of the teachings to reach the claimed invention would change the principles of operation of the devices shown in the respective cited art.

In the Office Action dated February 27, 2007, the Examiner states: "such a substitution would not change the "principles of operation" of the Sen et al. system.

But again, light and sound systems have different principles of operation. More particularly, Rees teaches, in the Abstract:

> Method and apparatus for detecting conditions in the atmosphere which are hazardous to flying aircraft and providing early warning to pilots or ground personnel. The method includes using a laser beam and a coherent optical receiver to optically sense sound waves produced by those hazardous conditions and measuring the effect of those sound waves on the transmitted and received optical beams.

Sen et al. teaches, in Fig. 9, "a detection system according to the present invention" (Col. 6, line 26) which shows a platform whereby an acoustic pulse generator 34 points downward and "sensor 33 has a tip that contacts the soil." Col. 6, line 36.

The Examiner's proposed attempt to combine teachings would change the principles of operation: As stated above, Rees is premised on the optical detection of the sound produced by weather (See Rees, as more precisely set out in Col. 1, line 39 etc., in view of the abstract) while Sen et al. uses a platform whereby an acoustic pulse generator 34 points downward so that "sensor 33 has a tip that contacts the soil." (See, e.g., Col. 6, line 36). Thus,

the transmitters, detectors, and other equipment of the respective patents operate by different principles of operation. For example, the beam splitters, an optical line array, telescope, etc. of Rees operate by different principles of operation than, e.g., Fig. 9, of Sen et al. For further example, receiving the "acoustic impulse" of Sen et al. in the "telescope" of Rees would not function without substantially changing principles of operation. Sen et al.'s detectors are unsuitable for optical detection (presumably, under the Examiner's theory, from the ground of Sen et al.).

Part of the reason that the proposed substitution is completely implausible is because the respective purposes are not the easiest to harmonize e.g., because one cannot pass a laser beam (Rees) underground (Sen) for the purpose of detecting "conditions in the atmosphere which are hazardous to flying aircraft" (Rees). Nor are weather conditions (Rees) to be found in the "granular bed" of Sen et al. Nor do non-metal land mines of Sen et al. fly in inclement weather (Rees) or require a ground crew. Nor does an acousto-optic mirror function in the granular bed of Sen et al. for detecting land mines. Contradiction between the respective references is pervasive, so of course their operating principles are incompatible and completely implausible.

It is the Examiner's burden to explain how these plainly different technologies could plausibly be combined without changing their operating principles, and the mere contention offered in the Office Action does not make it so.

Pursuant to 35 U.S.C. Sec. 132, the Examiner is required to explain how, pursuant to his rejection, the proposed combination or modification would not change the principles of operation - e.g., means for "detecting conditions in the atmosphere which are hazardous to flying aircraft and providing early warning to pilots or ground personnel / means for "detecting land mines with little or no metal." Representatively, the means for detecting in Rees "includes using a laser beam and a coherent optical receiver" and in some embodiments,

utilizes an acousto-optic mirror; the means for detecting in Sen et al. includes "sensor 33 [that] has a tip that contacts the soil." The Examiner's affidavit or declaration to substantiate the contention of fact that "such a substitution would not change the "principles of operation" of the Sen et al. system" is also required.

Applicant maintains that the contended teachings to reach the claimed invention would change the principles of operation of the devices shown in the respective cited art, and the rejection is improper for want of a *prima facie* showing of unpatentability.

4.   No motivation or suggestion has been shown in the cited art that, as of the date of the instant application, would have prompted one skilled in the art to make the combination to reach the claimed invention.

Applicant's Response notes that (4) no motivation or suggestion has been shown in the cited art that, as of the date of the instant application, would have prompted one skilled in the art to make the combination to reach the claimed invention.

In the Office Action dated February 27, 2007, the Examiner states: "the increased acoustic enhancement, as suggested by Rees, is motivation.

The Examiner has omitted that the reason for this feature in Rees is "to create an acousto-optic mirror" (Col. 15, line 60). An acousto-optic mirror does not function in the granular bed of Sen et al. for detecting land mines. Thus, the Examiner has provided no plausible reason to combine these diverse patents.

The Examiner's 35 U.S.C. Sec. 132 explanation, pursuant to his rejection, is required to justify the proposed reason to combine.

At least because of the extensive contradiction between these references of pervasively different technologies, no proper motivation has been cited to modify or combine the cited art to achieve the claimed invention. Thus, Applicant maintains that no motivation or suggestion has been shown in the prior art that, as of the date of the instant application, would have prompted one skilled in the art to make the combination or modification to reach the

claimed invention, and the rejection is improper for want of a *prima facie* showing of unpatentability.

5. Summary and Conclusion

In sum, the rejection is traversed as improper for many reasons, and reconsideration is requested. The rejection is improper because:

(1)   all claim requirements have not been shown in the cited art (e.g., no teaching of the claimed secondary wavelet, the receiving, and the processing as required in claim 1), thus no combination of the cited art can show that the claim as a whole was obvious;

(2)   the contended combination of teachings to reach the claimed invention would render the cited art inoperable for their respective purposes;

(3)   the contended combination of teachings to reach the claimed invention would change the principles of operation of the devices shown in the respective cited art; and

(4)   no motivation or suggestion has been shown in the cited art that, as of the date of the instant application, would have prompted one skilled in the art to make the combination or modification to reach the claimed invention.

For any and all of these reasons, the PTO has not shown *prima facie* obviousness, and withdrawal of the rejection is again requested, and otherwise the Sec. 132 explanations and the Examiner's declaration and affidavit have been required. If the rejection is not withdrawn, Applicant further requests an Interview with the Examiner and Supervising Primary Examiner.

In conclusion, Applicant wishes to make of record the fact that the Examiner is also the Examiner in Applicant's co-pending national case of PCT case PCT/US2004/039909, and attention is drawn to its U.S. National case Ser. No. 10/580,358.

III.   FEE

The Commissioner is hereby authorized to charge any fees associated with the

above-identified patent application or credit any overcharges to Deposit Account No. 50-0235.

Additionally, the Examiner is invited to contact the undersigned at (312) 240-0824 if it can in any

way expedite or ease the handling of this case. Please direct all correspondence to the

undersigned at the address given below.

Respectfully submitted,

Date:___April 26, 2007_____

Peter K. Trzyna
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(312) 240-0824

An Introduction to Wavelets

Abstract

Wavelets are mathematical functions that cut up data into different frequency components, and then study each component with a resolution matched to its scale. They have advantages over traditional Fourier methods in analyzing physical situations where the signal contains discontinuities and sharp spikes. Wavelets were developed independently in the fields of mathematics, quantum physics, electrical engineering, and seismic geology. Interchanges between these fields during the last ten years have led to many new wavelet applications such as image compression, turbulence, human vision, radar, and earthquake prediction. This paper introduces wavelets to the interested technical person outside of the digital signal processing field. I describe the history of wavelets beginning with Fourier, compare wavelet transforms with Fourier transforms, state properties and other special aspects of wavelets, and finish with some interesting applications such as image compression, musical tones, and de-noising noisy data.

Keywords: Wavelets, Signal Processing Algorithms, Orthogonal Basis Functions, Wavelet Applications

Contents:

$\mathcal{1}$

- o <u>Denoising Noisy Data</u>

- o <u>Detecting Self-Similarity in a Time Series</u>

- o <u>Musical Tones</u>

9. <u>Wavelets Endnote</u>

10. <u>References</u>

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2

Overview

The fundamental idea behind wavelets is to analyze according to scale. Indeed, some researchers in the wavelet field feel that, by using wavelets, one is adopting a whole new mindset or perspective in processing data.

Wavelets are functions that satisfy certain mathematical requirements and are used in representing data or other functions. This idea is not new. Approximation using superposition of functions has existed since the early 1800's, when Joseph Fourier discovered that he could superpose sines and cosines to represent other functions. However, in wavelet analysis, the *scale* that we use to look at data plays a special role. Wavelet algorithms process data at different *scales* or *resolutions*. If we look at a signal with a large "window," we would notice gross features. Similarly, if we look at a signal with a small "window," we would notice small features. The result in wavelet analysis is to see both the forest *and* the trees, so to speak.

This makes wavelets interesting and useful. For many decades, scientists have wanted more appropriate functions than the sines and cosines which comprise the bases of Fourier analysis, to approximate choppy signals (1). By their definition, these functions are non-local (and stretch out to infinity). They therefore do a very poor job in approximating sharp spikes. But with wavelet analysis, we can use approximating functions that are contained neatly in finite domains. Wavelets are well-suited for approximating data with sharp discontinuities.

The wavelet analysis procedure is to adopt a wavelet prototype function, called an *analyzing wavelet* or *mother wavelet*. Temporal analysis is performed with a contracted, high-frequency version of the prototype wavelet, while frequency analysis is performed with a dilated, low-frequency version of the same wavelet. Because the original signal or function can be represented in terms of a wavelet expansion (using coefficients in a linear combination of the wavelet functions), data operations can be performed using just the corresponding wavelet coefficients. And if you further choose the best wavelets adapted to your data, or truncate the coefficients below a threshold, your data is sparsely represented. This sparse coding makes wavelets an excellent tool in the field of data compression.

Other applied fields that are making use of wavelets include astronomy, acoustics, nuclear engineering, sub-band coding, signal and image processing, neurophysiology, music, magnetic resonance imaging, speech discrimination, optics, fractals, turbulence, earthquake-prediction, radar, human vision, and pure mathematics applications such as solving partial differential equations.

[Home Page] [Wavelet Page] [Contents] [Next Section]

3

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4

Historical Perspective

In the history of mathematics, wavelet analysis shows many different origins (2). Much of the work was performed in the 1930s, and, at the time, the separate efforts did not appear to be parts of a coherent theory.

Pre-1930

Before 1930, the main branch of mathematics leading to wavelets began with Joseph Fourier (1807) with his theories of frequency analysis, now often referred to as Fourier synthesis. He asserted that any 2π-periodic function $f(x)$ is the sum

$$a_0 + \sum_{k=1}^{\infty} \left(a_k \cos kx + b_k \sin kx \right) \quad (1)$$

of its Fourier series. The coefficients a_0, a_k, and b_k are calculated by

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(x)dx, \quad a_k = \frac{1}{\pi} \int_0^{2\pi} f(x)\cos(kx)dx, \quad b_k = \frac{1}{\pi} \int_0^{2\pi} f(x)\sin(kx)dx$$

Fourier's assertion played an essential role in the evolution of the ideas mathematicians had about the functions. He opened up the door to a new functional universe.

After 1807, by exploring the meaning of functions, Fourier series convergence, and orthogonal systems, mathematicians gradually were led from their previous notion of *frequency analysis* to the notion of *scale analysis*. That is, analyzing $f(x)$ by creating mathematical structures that vary in scale. How? Construct a function, shift it by some amount, and change its scale. Apply that structure in approximating a signal. Now repeat the procedure. Take that basic structure, shift it, and scale it again. Apply it to the same signal to get a new approximation. And so on. It turns out that this sort of scale analysis is less sensitive to noise because it measures the average fluctuations of the signal at different scales.

The first mention of wavelets appeared in an appendix to the thesis of A. Haar (1909). One property of the Haar wavelet is that it has *compact support,* which means that it vanishes outside of a finite interval. Unfortunately, Haar wavelets are not continuously differentiable which somewhat limits their applications.

The 1930s

In the 1930s, several groups working independently researched the representation of functions using *scale-varying basis functions*. Understanding the concepts of basis functions and scale-varying basis functions is key to understanding wavelets; the sidebar next provides a short detour lesson for those interested.

5

By using a scale-varying basis function called the Haar basis function (more on this later) Paul Levy, a 1930s physicist, investigated Brownian motion, a type of random signal (2). He found the Haar basis function superior to the Fourier basis functions for studying small complicated details in the Brownian motion.

Another 1930s research effort by Littlewood, Paley, and Stein involved computing the energy of a function $f(x)$:

$$energy = \frac{1}{2} \int_0^{2\pi} |f(x)|^2 dx \quad (2)$$

The computation produced different results if the energy was concentrated around a few points or distributed over a larger interval. This result disturbed the scientists because it indicated that energy might not be conserved. The researchers discovered a function that can vary in scale *and* can conserve energy when computing the functional energy. Their work provided David Marr with an effective algorithm for numerical image processing using wavelets in the early 1980s.

1960-1980

Between 1960 and 1980, the mathematicians Guido Weiss and Ronald R. Coifman studied the simplest elements of a function space, called *atoms,* with the goal of finding the atoms for a common function and finding the "assembly rules" that allow the reconstruction of all the elements of the function space using these atoms. In 1980, Grossman and Morlet, a physicist and an engineer, broadly defined wavelets in the context of quantum physics. These two researchers provided a way of thinking for wavelets based on physical intuition.

Post-1980

In 1985, Stephane Mallat gave wavelets an additional jump-start through his work in digital signal processing. He discovered some relationships between quadrature mirror filters, pyramid algorithms, and orthonormal wavelet bases (more on these later). Inspired in part by these results, Y. Meyer constructed the first non-trivial wavelets. Unlike the Haar wavelets, the Meyer wavelets are continuously differentiable; however they do not have compact support. A couple of years later, Ingrid Daubechies used Mallat's work to construct a set of wavelet orthonormal basis functions that are perhaps the most elegant, and have become the cornerstone of wavelet applications today.

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6

Sidebar- What are Basis Functions?

What are Basis Functions?

It is simpler to explain a basis function if we move out of the realm of analog (functions) and into the realm of digital (vectors) (*). Every two-dimensional vector (x,y) is a combination of the vector $(1,0)$ and $(0,1)$. These two vectors are the basis vectors for (x,y). Why? Notice that x multiplied by $(1,0)$ is the vector $(x,0)$, and y multiplied by $(0,1)$ is the vector $(0,y)$. The sum is (x,y).

The best basis vectors have the valuable extra property that the vectors are perpendicular, or orthogonal to each other. For the basis $(1,0)$ and $(0,1)$, this criteria is satisfied.

Now let's go back to the analog world, and see how to relate these concepts to basis functions. Instead of the vector (x,y), we have a function $f(x)$. Imagine that $f(x)$ is a musical tone, say the note A in a particular octave. We can construct A by adding sines and cosines using combinations of amplitudes and frequencies. The sines and cosines are the basis functions in this example, and the elements of Fourier synthesis. For the sines and cosines chosen, we can set the additional requirement that they be orthogonal. How? By choosing the appropriate combination of sine and cosine function terms whose inner product add up to zero. The particular set of functions that are orthogonal *and* that construct $f(x)$ are our orthogonal basis functions for this problem.

What are Scale-Varying Basis Functions?

A basis function varies in scale by chopping up the same function or data space using different scale sizes. For example, imagine we have a signal over the domain from 0 to 1. We can divide the signal with two step functions that range from 0 to 1/2 and 1/2 to 1. Then we can divide the original signal again using four step functions from 0 to 1/4, 1/4 to 1/2, 1/2 to 3/4, and 3/4 to 1. And so on. Each set of representations code the original signal with a particular resolution or scale.

Reference

(*) G. Strang, "Wavelets," American Scientist, Vol. 82, 1992, pp. 250-255.

[Home] [Wavelet Page] [Contents] [Previous] [Next]

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8

9

Fourier Analysis

Fourier's representation of functions as a superposition of sines and cosines has become ubiquitous for both the analytic and numerical solution of differential equations and for the analysis and treatment of communication signals. Fourier and wavelet analysis have some very strong links.

Fourier Transforms

The Fourier transform's utility lies in its ability to analyze a signal in the time domain for its frequency content. The transform works by first translating a function in the time domain into a function in the frequency domain. The signal can then be analyzed for its frequency content because the Fourier coefficients of the transformed function represent the contribution of each sine and cosine function at each frequency. An inverse Fourier transform does just what you'd expect, transform data from the frequency domain into the time domain.

Discrete Fourier Transforms

The discrete Fourier transform (DFT) estimates the Fourier transform of a function from a finite number of its sampled points. The sampled points are supposed to be typical of what the signal looks like at all other times.

The DFT has symmetry properties almost exactly the same as the continuous Fourier transform. In addition, the formula for the inverse discrete Fourier transform is easily calculated using the one for the discrete Fourier transform because the two formulas are almost identical.

Windowed Fourier Transforms

If $f(t)$ is a nonperiodic signal, the summation of the periodic functions, sine and cosine, does not accurately represent the signal. You could artificially extend the signal to make it periodic but it would require additional continuity at the endpoints. The windowed Fourier transform (WFT) is one solution to the problem of better representing the nonperiodic signal. The WFT can be used to give information about signals simultaneously in the time domain and in the frequency domain.

With the WFT, the input signal $f(t)$ is chopped up into sections, and each section is analyzed for its frequency content separately. If the signal has sharp transitions, we window the input data so that the sections converge to zero at the endpoints (3). This windowing is accomplished via a weight function that places less emphasis near the interval's endpoints than in the middle. The effect of the window is to localize the signal in time.

10

Fast Fourier Transforms

To approximate a function by samples, and to approximate the Fourier integral by the discrete Fourier transform, requires applying a matrix whose order is the number sample points n. Since multiplying an $n \times n$ matrix by a vector costs on the order of n^2 arithmetic operations, the problem gets quickly worse as the number of sample points increases. However, if the samples are uniformly spaced, then the Fourier matrix can be factored into a product of just a few sparse matrices, and the resulting factors can be applied to a vector in a total of order $n \log n$ arithmetic operations. This is the so-called *fast Fourier transform* or FFT (4).

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$\mathbf{W}$avelet vs $\mathbf{F}$ourier $\mathbf{T}$ransforms

Similarities between Fourier and Wavelet Transforms

The fast Fourier transform (FFT) and the discrete wavelet transform (DWT) are both linear operations that generate a data structure that contains $\log_2 n$ segments of various lengths, usually filling and transforming it into a different data vector of length 2^*.

The mathematical properties of the matrices involved in the transforms are similar as well. The inverse transform matrix for both the FFT and the DWT is the transpose of the original. As a result, both transforms can be viewed as a rotation in function space to a different domain. For the FFT, this new domain contains basis functions that are sines and cosines. For the wavelet transform, this new domain contains more complicated basis functions called wavelets, mother wavelets, or analyzing wavelets.

Both transforms have another similarity. The basis functions are localized in frequency, making mathematical tools such as power spectra (how much power is contained in a frequency interval) and scalegrams (to be defined later) useful at picking out frequencies and calculating power distributions.

Dissimilarities between Fourier and Wavelet Transforms

The most interesting dissimilarity between these two kinds of transforms is that individual wavelet functions are *localized in space.* Fourier sine and cosine functions are not. This localization feature, along with wavelets' localization of frequency, makes many functions and operators using wavelets "sparse" when transformed into the wavelet domain. This sparseness, in turn, results in a number of useful applications such as data compression, detecting features in images, and removing noise from time series.

One way to see the time-frequency resolution differences between the Fourier transform and the wavelet transform is to look at the basis function coverage of the time-frequency plane (5). Figure 1 shows a windowed Fourier transform, where the window is simply a square wave. The square wave window truncates the sine or cosine function to fit a window of a particular width. Because a single window is used for all frequencies in the WFT, the resolution of the analysis is the same at all locations in the time-frequency plane.
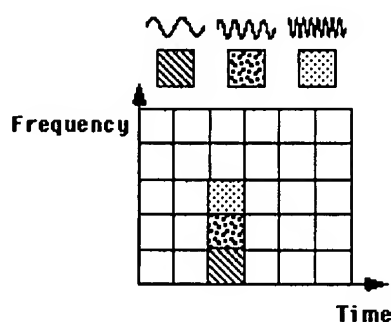
12

Fig. 1. Fourier basis functions, time-frequency tiles, and coverage of the time-frequency plane.

An advantage of wavelet transforms is that the windows *vary*. In order to isolate signal discontinuities, one would like to have some very short basis functions. At the same time, in order to obtain detailed frequency analysis, one would like to have some very long basis functions. A way to achieve this is to have short high-frequency basis functions and long low-frequency ones. This happy medium is exactly what you get with wavelet transforms. Figure 2 shows the coverage in the time-frequency plane with one wavelet function, the Daubechies wavelet.
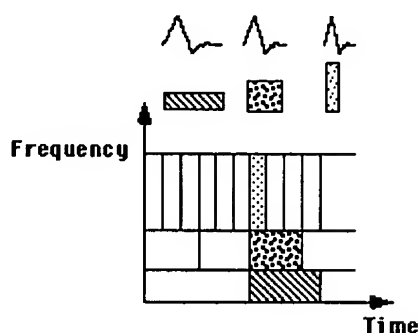
Fig. 2. Daubechies wavelet basis functions, time-frequency tiles, and coverage of the time-frequency plane.

One thing to remember is that wavelet transforms do not have a single set of basis functions like the Fourier transform, which utilizes just the sine and cosine functions. Instead, wavelet transforms have an infinite set of possible basis functions. Thus wavelet analysis provides immediate access to information that can be obscured by other time-frequency methods such as Fourier analysis.

[Home] [Wavelet Page] [Contents] [Previous] [Next]

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13

14

What do Some Wavelets Look Like?

Wavelet transforms comprise an infinite set. The different wavelet families make different trade-offs between how compactly the basis functions are localized in space and how smooth they are.

Some of the wavelet bases have fractal structure. The Daubechies wavelet family is one example (see Figure 3).
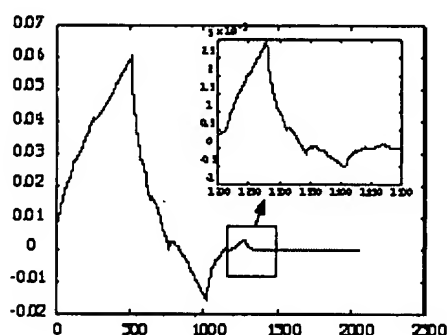

Fig. 3. The fractal self-similiarity of the Daubechies mother wavelet. This figure was generated using the WaveLab command: `wave=MakeWavelet(2, -4, 'Daubechies', 4, 'Mother', 2048)`. The inset figure was created by zooming into the region x=1200 to 1500.

Within each family of wavelets (such as the Daubechies family) are wavelet subclasses distinguished by the number of coefficients and by the level of iteration. Wavelets are classified within a family most often by the *number of vanishing moments*. This is an extra set of mathematical relationships for the coefficients that must be satisfied, and is directly related to the number of coefficients (1). For example, within the Coiflet wavelet family are Coiflets with two vanishing moments, and Coiflets with three vanishing moments. In Figure 4, I illustrate several different wavelet families.
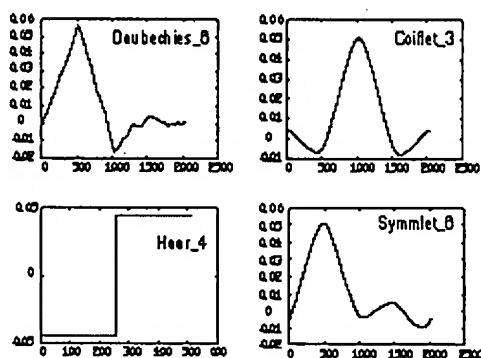

Fig. 4. Several different families of wavelets. The number next to the wavelet name

15

represents the number of vanishing moments (A stringent mathematical definition related to the number of wavelet coefficients) for the subclass of wavelet. Note: These figures were created using <u>WaveLab,</u> by typing:

```
wave = MakeWavelet(2,-4,'Daubechies',6,'Mother', 2048);
wave = MakeWavelet(2,-4,'Coiflet',3,'Mother', 2048);
wave = MakeWavelet(0,0,'Haar',4,'Mother', 512);
wave = MakeWavelet(2,-4,'Symmlet',6,'Mother', 2048);
```

[Home] [Wavelet Page] [Contents] [Previous] [Next]

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16

Wavelet Analysis

Now we begin our tour of wavelet theory, when we analyze our signal in time for its frequency content. Unlike Fourier analysis, in which we analyze signals using sines and cosines, now we use wavelet functions.

The Discrete Wavelet Transform

Dilations and translations of the "Mother function," or "analyzing wavelet" $\Phi(x)$ define an orthogonal basis, our wavelet basis:

$$\Phi_{(s,l)}(x) = 2^{\frac{-s}{2}}\Phi\left(2^{-s}x - l\right) \quad (3)$$

The variables s and l are integers that scale and dilate the mother function $\Phi(x)$ to generate wavelets, such as a Daubechies wavelet family. The scale index s indicates the wavelet's width, and the location index l gives its position. Notice that the mother functions are rescaled, or "dilated" by powers of two, and translated by integers. What makes wavelet bases especially interesting is the self-similarity caused by the scales and dilations. Once we know about the mother functions, we know everything about the basis.

To span our data domain at different resolutions, the analyzing wavelet is used in a scaling equation:

$$W(x) = \sum_{k=-1}^{N-2}(-1)^k c_{k+1}\Phi(2x + k) \quad (4)$$

where $W(x)$ is the scaling function for the mother function $\Phi(x)$, and c_k are the *wavelet coefficients*. The wavelet coefficients must satisfy linear and quadratic constraints of the form

$$\sum_{k=0}^{N-1} c_k = 2 \quad , \quad \sum_{k=0}^{N-1} c_k c_{k+2l} = 2\delta_{l,0}$$

where δ is the delta function and l is the location index.

One of the most useful features of wavelets is the ease with which a scientist can choose the defining coefficients for a given wavelet system to be adapted for a given problem. In Daubechies' original paper (6), she developed specific families of wavelet systems that were very good for representing polynomial behavior. The Haar wavelet is even simpler, and it is often used for educational purposes.

It is helpful to think of the coefficients $\{c_0,....,c_n\}$ as a filter. The filter or coefficients are placed in a transformation matrix, which is applied to a raw data vector. The coefficients are

ordered using two dominant patterns, one that works as a smoothing filter (like a moving average), and one pattern that works to bring out the data's "detail" information. These two orderings of the coefficients are called a *quadrature mirror filter pair* in signal processing parlance. A more detailed description of the transformation matrix can be found elsewhere (4).

To complete our discussion of the DWT, let's look at how the wavelet coefficient matrix is applied to the data vector. The matrix is applied in a hierarchical algorithm, sometimes called a *pyramidal algorithm*. The wavelet coefficients are arranged so that odd rows contain an ordering of wavelet coefficients that act as the smoothing filter, and the even rows contain an ordering of wavelet coefficient with different signs that act to bring out the data's detail. The matrix is first applied to the original, full-length vector. Then the vector is smoothed and decimated by half and the matrix is applied again. Then the smoothed, halved vector is smoothed, and halved again, and the matrix applied once more. This process continues until a trivial number of "smooth-smooth-smooth..." data remain. That is, each matrix application brings out a higher resolution of the data while at the same time smoothing the remaining data. The output of the DWT consists of the remaining "smooth (etc.)" components, and all of the accumulated "detail" components.

The Fast Wavelet Transform

The DWT matrix is not sparse in general, so we face the same complexity issues that we had previously faced for the discrete Fourier transform (7). We solve it as we did for the FFT, by factoring the DWT into a product of a few sparse matrices using self-similarity properties. The result is an algorithm that requires only order n operations to transform an n-sample vector. This is the "fast" DWT of Mallat and Daubechies.

Wavelet Packets

The wavelet transform is actually a subset of a far more versatile transform, the wavelet packet transform (8).

Wavelet packets are particular linear combinations of wavelets (7). They form bases which retain many of the orthogonality, smoothness, and localization properties of their parent wavelets. The coefficients in the linear combinations are computed by a recursive algorithm making each newly computed wavelet packet coefficient sequence the root of its own analysis tree.

Adapted Waveforms

Because we have a choice among an infinite set of basis functions, we may wish to find the best basis function for a given representation of a signal (7). A *basis of adapted waveform* is the best basis function for a given signal representation. The chosen basis carries substantial information about the signal, and if the basis description is efficient (that is, very few terms in the expansion are needed to represent the signal), then that signal information has been compressed.

According to Wickerhauser (7), some desirable properties for adapted wavelet bases are

1. speedy computation of inner products with the other basis functions;

18

2. speedy superposition of the basis functions;
3. good spatial localization, so researchers can identify the position of a signal that is contributing a large component;
4. good frequency localization, so researchers can identify signal oscillations; and
5. independence, so that not too many basis elements match the same portion of the signal.

For adapted waveform analysis, researchers seek a basis in which the coefficients, when rearranged in decreasing order, decrease as rapidly as possible. to measure rates of decrease, they use tools from classical harmonic analysis including calculation of *information cost functions*. This is defined as the expense of storing the chosen representation. Examples of such functions include the number above a threshold, concentration, entropy, logarithm of energy, Gauss-Markov calculations, and the theoretical dimension of a sequence.

[Home] [Wavelet Page] [Contents] [Previous] [Next]

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Computer and Human Vision

In the early 1980s, David Marr began work at MIT's Artificial Intelligence Laboratory on artificial vision for robots. He is an expert on the human visual system and his goal was to learn why the first attempts to construct a robot capable of understanding its surroundings were unsuccessful (2).

Marr believed that it was important to establish scientific foundations for vision, and that while doing so, one must limit the scope of investigation by excluding everything that depends on training, culture, and so on, and focus on the mechanical or involuntary aspects of vision. This low-level vision is the part that enables us to recreate the three-dimensional organization of the physical world around us from the excitations that stimulate the retina. Marr asked the questions:

- How is it possible to define the contours of objects from the variations of their light intensity?
- How is it possible to sense depth?
- How is movement sensed?

He then developed working algorithmic solutions to answer each of these questions.

Marr's theory was that image processing in the human visual system has a complicated hierarchical structure that involves several layers of processing. At each processing level, the retinal system provides a visual representation that scales progressively in a geometrical manner. His arguments hinged on the detection of intensity changes. He theorized that intensity changes occur at different scales in an image, so that their optimal detection requires the use of operators of different sizes. He also theorized that sudden intensity changes produce a peak or trough in the first derivative of the image. These two hypotheses require that a vision filter have two characteristics: it should be a differential operator, and it should be capable of being tuned to act at any desired scale. Marr's operator was a wavelet that today is referred to as a "Marr wavelet."

[Home] [Wavelet Page] [Contents] [Previous] [Next]

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20

F BI Fingerprint Compression

Between 1924 and today, the US Federal Bureau of Investigation has collected about 30 million sets of fingerprints (7). The archive consists mainly of inked impressions on paper cards. Facsimile scans of the impressions are distributed among law enforcement agencies, but the digitization quality is often low. Because a number of jurisdictions are experimenting with digital storage of the prints, incompatibilities between data formats have recently become a problem. This problem led to a demand in the criminal justice community for a digitization and a compression standard.

In 1993, the FBI's Criminal Justice Information Services Division developed standards for fingerprint digitization and compression in cooperation with the National Institute of Standards and Technology, Los Alamos National Laboratory, commercial vendors, and criminal justice communities (9).

Let's put the data storage problem in perspective. Fingerprint images are digitized at a resolution of 500 pixels per inch with 256 levels of gray-scale information per pixel. A single fingerprint is about 700,000 pixels and needs about 0.6 Mbytes to store. A pair of hands, then, requires about 6 Mbytes of storage. So digitizing the FBI's current archive would result in about 200 terabytes of data. (Notice that at today's prices of about $900 per Gbyte for hard-disk storage, the cost of storing these uncompressed images would be about a 200 million dollars.) Obviously, data compression is important to bring these numbers down.

Fig. 5. An FBI-digitized left thumb fingerprint. The image on the left is the original; the one on the right is reconstructed from a 26:1 compression. These images can be retrieved by anonymous FTP. (Courtesy Chris Brislawn, Los Alamos National Laboratory.

[Home] [Wavelet Page] [Contents] [Previous] [Next]

21

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22

Denoising Noisy Data

In diverse fields from planetary science to molecular spectroscopy, scientists are faced with the problem of recovering a true signal from incomplete, indirect or noisy data. Can wavelets help solve this problem? The answer is certainly "yes," through a technique called *wavelet shrinkage and thresholding methods,* that David Donoho has worked on for several years (10).

The technique works in the following way. When you decompose a data set using wavelets, you use filters that act as *averaging* filters and others that produce *details* (11). Some of the resulting wavelet coefficients correspond to details in the data set. If the details are small, they might be omitted without substantially affecting the main features of the data set. The idea of *thresholding,* then, is to set to zero all coefficients that are less than a particular threshold. These coefficients are used in an inverse wavelet transformation to reconstruct the data set. Figure 6 is a pair of "before" and "after" illustrations of a nuclear magnetic resonance (NMR) signal. The signal is transformed, thresholded and inverse-transformed. The technique is a significant step forward in handling noisy data because the denoising is carried out without smoothing out the sharp structures. The result is cleaned-up signal that still shows important details.
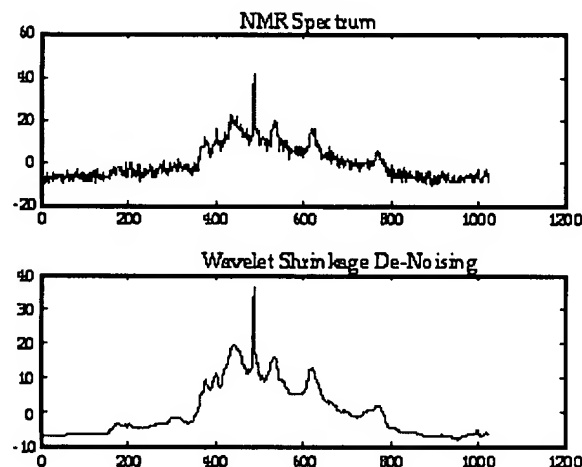
Fig. 6. "Before" and "after" illustrations of a nuclear magnetic resonance signal. The original signal is at the top, the denoised signal at the bottom. (Images courtesy David Donoho, Stanford University, NMR data courtesy Adrian Maudsley, VA Medical Center, San Francisco).

Figure 7 displays an image created by Donoho of Ingrid Daubechies (an active researcher in wavelet analysis and the inventor of smooth orthonormal wavelets of compact support), and then several close-up images of her eye: an original, an image with noise added, and finally denoised image. To denoise the image, Donoho:

23

1. transformed the image to the wavelet domain using Coiflets with three vanishing moments,
2. applied a threshold at two standard deviations, and
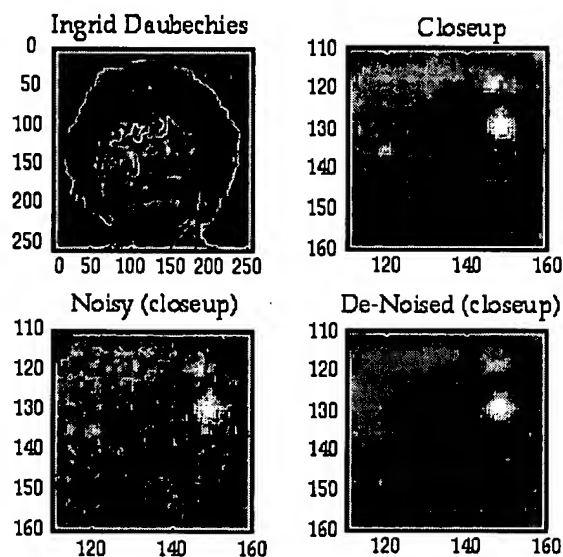3. inverse-transformed the image to the signal domain.

Fig. 7. Denoising an image of Ingrid Daubechies' left eye. The top left image is the original. At top right is a close-up image of her left eye. At bottom left is a close-up image with noise added. At bottom right is a close-up image, denoised. The photograph of Daubechies was taken at the 1993 AMS winter meetings with a Canon XapShot video still-frame camera. (Courtesy David Donoho)

[Home] [Wavelet Page] [Contents] [Previous] [Next]

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Detecting Self-similarity in a Time Series

Wavelet analysis is proving to be a very powerful tool for characterizing behavior, especially self-similar behavior, over a wide range of time scales.

In 1993, Scargle and colleagues at NASA-Ames Research Center and elsewhere investigated the quasiperiodic oscillations (QPOs) and very low-frequency noise (VLFN) from an astronomical X-ray accretion source, Sco X-1 as possibly being caused by the same physical phenomenon (12). Sco X-1 is part of a close binary star system in which one member is a late main sequence star and the other member (Sco X-1) is a compact star generating bright X rays. The causes for QPOs in X-ray sources have been actively investigated in the past, but other aperiodic phenomena such as VLFNs have not been similarly linked in the models. Their Sco X-1 data set was an interesting 5-20 keV EXOSAT satellite time-series consisting of a wide-range of time scales, from 2 ms to almost 10 hours.

Galactic X-ray sources are often caused by the accretion of gas from one star to another in a binary star system. The accreted object is usually a compact star such as a white dwarf, neutron star, or black hole. Gas from the less massive star flows to the other star via an accretion disk (that is, a disk of matter around the compact star flowing inward) around the compact star. The variable luminosities are caused by irregularities in the gas flow. The details of the gas flow are not well-known.

The researchers noticed that the luminosity of Sco X-1 varied in a self-similar manner, that is, the statistical character of the luminosities examined at different time resolutions remained the same. Since one of the great strengths of wavelets is that they can process information effectively at different scales, Scargle used a wavelet tool called a scalegram to investigate the time-series.

Scargle defines a *scalegram* of a time series as the average of the squares of the wavelet coefficients at a given scale. Plotted as a function of scale, it depicts much of the same information as does the Fourier power spectrum plotted as a function of frequency. Implementing the scalegram involves summing the product of the data with a wavelet function, while implementing the Fourier power spectrum involves summing the data with a sine or cosine function. The formulation of the scalegram makes it a more convenient tool than the Fourier transform because certain relationships between the different time scales become easier to see and correct, such as seeing and correcting for photon noise.

The scalegram for the time-series clearly showed the QPOs and the VLFNs, and the investigators were able to calculate a power-law to the frequencies. Subsequent simulations suggested that the cause of Sco-X1's luminosity fluctuations may be due to a chaotic accretion flow.

25

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26

Musical Tones

Victor Wickerhauser has suggested that wavelet packets could be useful in sound synthesis (13). His idea is that a single wavelet packet generator could replace a large number of oscillators. Through experimentation, a musician could determine combinations of wave packets that produce especially interesting sounds.

Wickerhauser feels that sound synthesis is a natural use of wavelets. Say one wishes to approximate the sound of a musical instrument. A sample of the notes produced by the instrument could be decomposed into its wavelet packet coefficients. Reproducing the note would then require reloading those coefficients into a wavelet packet generator and playing back the result. Transient characteristics such as attack and decay- roughly, the intensity variations of how the sound starts and ends- could be controlled separately (for example, with envelope generators), or by using longer wave packets and encoding those properties as well into each note. Any of these processes could be controlled in real time, for example, by a keyboard.

Notice that the musical instrument could just as well be a human voice, and the notes words or phonemes.

A wavelet-packet-based music synthesizer could store many complex sounds efficiently because

- wavelet packet coefficients, like wavelet coefficients, are mostly very small for digital samples of smooth signals; and
- discarding coefficients below a predetermined cutoff introduces only small errors when we are compressing the data for smooth signals.

Similarly, a wave packet-based speech synthesizer could be used to reconstruct highly compressed speech signals. Figure 8 illustrates a wavelet musical tone or *toneburst*.

Fig. 8. Wavelets for music: a graphical representation of a Wickerhauser toneburst. This screenshot of a toneburst was taken while it was playing in the Macintosh commercial sound program Kaboom! Factory. (Toneburst courtesy Victor Wickerhauser)

Press 🔊 to hear this wavelet toneburst (110 kB).

27

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Endnote

Most of basic wavelet theory has been done. The mathematics have been worked out in excruciating detail and wavelet theory is now in the refinement stage. The refinement stage involves generalizations and extensions of wavelets, such as extending wavelet packet techniques.

The future of wavelets lies in the as-yet uncharted territory of *applications*. Wavelet techniques have not been thoroughly worked out in applications such as practical data analysis, where for example discretely sampled time-series data might need to be analyzed. Such applications offer exciting avenues for exploration.

[Home] [Wavelet Page] [Contents] [Previous] [Next]

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29

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30

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Some Wavelet Resources

The amount of wavelets-related material is multiplying. Many papers and software sources are on Internet.

- Press [image] for my current list of **Wavelet Resources**.

About the Author

(updated January 2003) Amara Graps is an astronomer and consultant working on infrared space physics, interplanetary dust dynamics, popular science writing, and numerical analysis projects for companies as well as for hospitals, government laboratories, and universities. Her work experience, primarily in astronomy, astrophysics, and planetary science research, was gained from her current research at IFSI, her previous research at the Max-Planck-Institut für Kernphysik [MPI-K], Heidelberg, Germany, and jobs at Stanford University, NASA-Ames, the University of Colorado, and the Jet Propulsion Laboratory. She earned her B.S. in Physics in 1984 from the University of California, Irvine, her M.S. in Physics (w/computational physics option) in 1991 from San Jose State University, and her PhD in physics from the University of Heidelberg and MPI-K.

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[Home] [Wavelet Page] [Contents] [Previous]

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31

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32

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of a belt-driven metal-turning lathe. It consists of a simple layshaft, which may be brought into gear with the coned pulley and mandrel when required.

background (*Phys*) Extraneous signals arising from any cause which might be confused with the required measurements, eg in electrical measurements of nuclear phenomena and of radioactivity, it would include counts emanating from amplifier noise, cosmic rays and insulator leakage. Cf **signal-to-noise ratio**.

background job (*Comp*) Job having a low priority within a multiprogramming system. See **foreground/background processing, job queue, time sharing.**

background noise (*Acous*) Extraneous noise contaminating sound measurements and which cannot be separated from wanted signals. Residual output from eg microphones, pickups, lines, giving a **signal-to-noise ratio.** Also *ground noise*.

background radiation (*Astron*) See **microwave background.**

background radiation (*Radiol*) Radiation coming from sources other than that being observed.

background video (*ImageTech*) A technique for overlaying video on previously recorded depth multiplex audio. Abbrev *BCV*. Also *video on sound* (abbrev *VOS*). See **depth multiplex recording.**

backhand welding (*Eng*) Welding in which the torch or electrode hand faces the direction of travel, thus postheating the existing weld. Cf **forehand welding.**

backheating (*Electronics*) Excess heating of a cathode due to bombardment by high-energy electrons returning to the cathode. In magnetrons, it may be sufficient to keep the cathode at operating temperature without external heating.

backing (*ImageTech*) Light-absorbent layer on the rear surface of photographic film or plate to reduce halation.

backing (*Meteor*) The changing of a wind in a counterclockwise direction. Cf **veering.**

backing (*Print*) The binding process by which one half of the sections at the back of a volume are bent over to the right and the other half to the left. The projections formed are joints, to which the case is hinged, by hand or machine. Some machines also simultaneously perform rounding.

backing boards (*Print*) Wedge-shaped wooden boards between which an unbound book is held in the lyingpress, while the joints are being formed for attaching the case.

backings (*Build*) Wooden battens, secured to rough walls, for the fixing of wood linings etc. Also *strapping*.

backing store (*Comp*) Means of storing large amounts of data outside the main memory. Will be a combination of **magnetic disk, magnetic drum, magnetic tape, optical disk,** containing archive files, **back-up files.** Also *secondary memory.*

backing-up (*Build*) The use of inferior bricks for the inner face of a wall.

backing-up (*Print*) (1) Printing on the second side of a sheet. (2) Backing a letterpress printing plate to required height.

back inlet gulley (*Build*) A trapped gulley in which the inlets discharge under the grating and above the level of the water in the trap, so that splashing and blocking of the grating are avoided.

back iron (*Build*) The stiffening plate screwed to the cutting iron of a plane. Also *cap iron, cover iron.*

back joint (*Build*) The part of the back of a stone step which is dressed to fit into the rebate of the upper step.

back-kick (*Eng*) The violent reversal of an internal-combustion engine during starting, due to a back-fire.

backlash (*Eng*) The lost motion between two elements of mechanism, ie the amount the first has to move, owing to imperfect connection, before communicating its motion to the second.

backlash (*Telecomm*) (1) Mechanical deficiency in a tuning control, with a difference in dial reading between clockwise and anti-clockwise rotation. (2) Property of most re-

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generative and oscillator circuits, by which oscillation is maintained with a smaller positive feedback than is required for inception.

backlight (*Electronics*) The light source (often a cold cathode discharge in a flat fluorescent envelope) used in some light-modulating flat panel displays such as those based on liquid crystals. See panel on **Liquid crystal display.**

backlight compensation (*ImageTech*) The opening of the iris to correctly expose a backlit subject which would otherwise be a silhouette. Abbrev *BLC.*

back lighting (*ImageTech*) Lighting illuminating the subject from behind, opposite the camera, often to provide rim light or halo effects.

back lining (*Print*) See **hollows.**

back lobe (*Telecomm*) Lobe of polar diagram for antenna, microphone etc which points in the reverse direction to that required.

backlocking (*CivEng*) Holding a signal lever partially restored until completion of a predetermined sequence of operations.

backmatter (*Print*) US for end matter.

back-mixing (*ChemEng*) In a chemical reactor, jet engine or other apparatus through which material flows and thereby undergoes a change in some property, the turning back of some of the material so that it mixes with that which has entered the reactor after it.

back-mutation (*Biol*) See **reversion.**

back observation (*Surv*) An observation made with instrument on station just left. Also *back sight.*

back-off (*MinExt*) (1) To raise the drilling bit or down-hole assembly for a short distance from the bottom of the hole. (2) To unscrew drilling components.

back porch (*ImageTech*) A short period of black level signal transmitted at the end of the horizontal sync pulse before the picture information.

back-porch effect (*Telecomm*) The prolonging of the collector current in a transistor for a brief time after the input signal (particularly if large) has decreased to zero.

back pressure (*Build*) Air pressure in drainage pipes exceeding atmospheric pressure.

back pressure (*Eng*) (1) The pressure opposing the motion of the piston of an engine on its exhaust stroke. (2) The exhaust pressure of a turbine. Increased by clogged or defective exhaust system.

back pressure (*Med*) Proximal pressure produced by an obstruction to fluid flowing through the cardio-vascular or urinary systems.

back pressure (*MinExt*) The hydrostatic pressure exerted by the fluids in the bore of a well which acts against that of the oil and gas in the strata. Control of this pressure by valves and chokes maintains an even and productive flow of oil.

back-pressure turbine (*Eng*) A steam turbine from which the whole of the exhaust steam, at a suitable pressure, is taken for heating purposes.

back projection (*ImageTech*) (1) Projection of a picture, from film, transparency or video, on to a translucent screen to be viewed from the opposite side. (2) A form of motion picture composite photography in which the projected picture forms the background to action taking place in front of it, both being photographed together.

back rake (*Eng*) In a lathe tool, the inclination of the top surface or face to a plane parallel to the base of the tool.

back saw (*Build*) A saw stiffened by a thickened back, eg a tenon saw.

back scatter (*Phys*) The deflection of radiation or particles by scattering through angles greater than 90° with reference to the original direction of travel. Cf **forward scatter.**

backcutting (*Build*) A stone with a boasted or broached face, but with smooth border all round.

back shore (*Build*) One of the outer members of an arrangement of raking shores or props, for supporting temporarily the side of a building. The back shore supports the raking shore, or raker, which takes the thrust from the highest part of the building.

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Third Edition

photographic film is placed between the source of x-rays and the crystal specimen.

back resistance [ELECTR] The resistance between the contacts opposing the inverse current of a metallic rectifier.

backrope [NAV ARCH] 1. Either of two ropes or chains on a sailing ship, extending aft from the lower end of the dolphin striker to each side of the bows. 2. *See* cat back.

back-run process [CHEM ENG] A process for manufacturing water gas in which part of the run is made down, by passing steam through the superheater, thence up through the carburetor, down through the generator, and direct to the scrubbers.

back rush [OCEANOGR] Return of water seaward after the uprush of the waves.

backs [MIN ENG] Ore height available above a given working level.

backsaw [DES ENG] A fine-tooth saw with its upper edge stiffened by a metal rib to ensure straight cuts.

backsawing [FOR] A method of converting timber so that the growth rings meet the face in any part of an angle of less than 45°. Also known as bastard-sawing; crown-cut; plain-sawing; slash-sawing.

backscatter gage [ENG] A radar instrument used to measure the radiation scattered at 180° to the direction of the incident wave.

backscattering Also known as back radiation; backward scattering. [COMMUN] Propagation of extraneous signals by F or E-region reflection in addition to the desired ionospheric scatter mode; the undesired signal enters the antenna through the back lobes. [ELECTROMAG] 1. Radar echoes from a target. 2. Undesired radiation of energy to the rear by a directional antenna. [PHYS] The deflection of radiation or nuclear particles by scattering processes through angles greater than 90° with respect to the original direction of travel.

backscattering thickness gage [ENG] A device that uses a radioactive source for measuring the thickness of materials, such as coatings, in which the source and the instrument measuring the radiation are mounted on the same side of the material, the backscattered radiation thus being measured.

backscatter radar [ORD] An over-the-horizon radar system designed primarily to detect an enemy missile attack from far beyond the horizon, by detecting radio energy reflected from the ionosphere disturbance created by the missiles.

backset [BUILD] The horizontal distance from the face of a lock or latch to the center of the keyhole, knob, or lock cylinder.

back-set bed [GEOL] Cross bedding that dips in a direction against the flow of a depositing current.

backshore [GEOL] The upper shore zone that is beyond the advance of the usual waves and tides. Also known as back beach; backshore beach.

backshore beach *See* backshore.

backshore terrace *See* berm.

back shot [MIN ENG] A shot used for widening an entry, placed at some distance from the head of an entry.

backsight [ENG] 1. A sight on a previously established survey point or line. 2. Reading a leveling rod in its unchanged position after moving the leveling instrument to a different location. [NAV] A marine sextant observation of a celestial body made by facing 180° from the azimuth of the body and using the visible horizon in the direction in which the observer is facing.

backsight method [ENG] 1. A plane-table traversing method in which the table orientation produces the alignment of the alidade on an established map line, the table being rotated until the line of sight is coincident with the corresponding ground line. 2. Sighting two pieces of equipment directly at each other in order to orient and synchronize one with the other in azimuth and elevation.

back siphonage [CIV ENG] The flowing back of used, contaminated, or polluted water from a plumbing fixture or vessel into the pipe which feeds it; caused by reduced pressure in the pipe.

back slope *See* dip slope.

backspace [COMPUT SCI] To move a recording medium one unit in the reverse or background direction. [MECH ENG] To move a typewriter carriage back one space by depressing a back space key.

backspring [NAV ARCH] A heavy line extending forward at an acute angle with a ship from the stern or midships to a wharf.

backstay [ENG] 1. A supporting cable that prevents a more or less vertical object from falling forward. 2. A spring used to

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keep together the cutting edges of purchase shears that runs from either end of a carriage's rear axle to A leather strip that covers and strengthens a shoo [GRAPHICS] A rope or strap that keeps the carriage printing press from moving too far forward. [NAV rope, wire, or cable that runs from the top of a mast a ship and slants a little aft. [TEXT] A bar having on its top, that runs across a loom beneath the lower the warp yarns.

backstop sequence [MET] Sequential deposit beads in the direction opposite to the direction of **back stoping** *See* shrinkage stoping.

backswamp [GEOL] Swampy depressed area of between the natural levees and the edge of the flood

backswamp depression [ECOL] A low swamp cent to river levees.

back sweetening [CHEM ENG] The controlled commercial-grade mercaptans to a petroleum stock cess free sulfur in order to reduce free sulfur by disulfide.

backtalk [COMPUT SCI] Passage of information standby computer to the active computer.

backthrusting [GEOL] The thrusting in the direct interior of an orogenic belt, opposite the general trend.

back titration [CHEM] A titration to return to the which was passed.

back-to-front ratio [ELECTROMAG] Ratio used tion with an antenna, metal rectifier, or any device signal strength or resistance in one direction is comp that in the opposite direction.

backtracking [COMPUT SCI] A method of solving automatically by a systematic search of the possible invalid solutions are eliminated and are not retried.

backup [BUILD] That part of a masonry wall be exterior facing. [CIV ENG] Overflow in a drain system, due to stoppage. [ENG] 1. An item under ment intended to perform the same general function other item also under development performs. 2. A comp material used behind a sealant to reduce its depth and to the sealant against sag or indentation. [GRAPHICS] image printed on the reverse side of a printed sheet. printing of such an image. [MET] A support used to the upsetting force in the workpieces during flash [PETRO ENG] During drilling, the holding of one pipe while another is screwed out of it or into it.

backup arrangement *See* cascade.

backup relay [ELEC] A relay designed to protect a system in case a primary relay fails to operate as desi

backup strip [BUILD] A wood strip which is fast corner of a partition or wall to provide a nailing surface of lath. Also known as lathing board.

backup system [SYS ENG] A system, normally but kept available to replace a system which may operation.

backup tong [ENG] A heavy device used on a drill loosen the tool joints.

Backus-Naur form [COMPUT SCI] A metalanguage specifies which sequences of symbols constitute a syntax valid program language. Abbreviated BNF.

back veneer [MATER] In veneer plywood, the layer neer on the side of a plywood sheet which is opposite veneer; usually of lower quality.

back vent [CIV ENG] An individual vent for a plu fixture located on the downstream (sewer) side of a t protect the trap against siphonage.

backward-acting regulator [ELECTR] Transmissio ulator in which the adjustment made by the regulator affe quantity which caused the adjustment.

backward-bladed aerodynamic fan [MECH ENG] that consists of several streamlined blades mounted in a r ing casing.

backward diode [ELECTR] A semiconductor diode to a tunnel diode except that it has no forward tunnel cu used as a low-voltage rectifier.

backward error analysis [COMPUT SCI] A form of analysis which seeks to replace all errors made in the co solving a problem by an equivalent perturbation of the ori problem.

backward folding *See* backfolding.